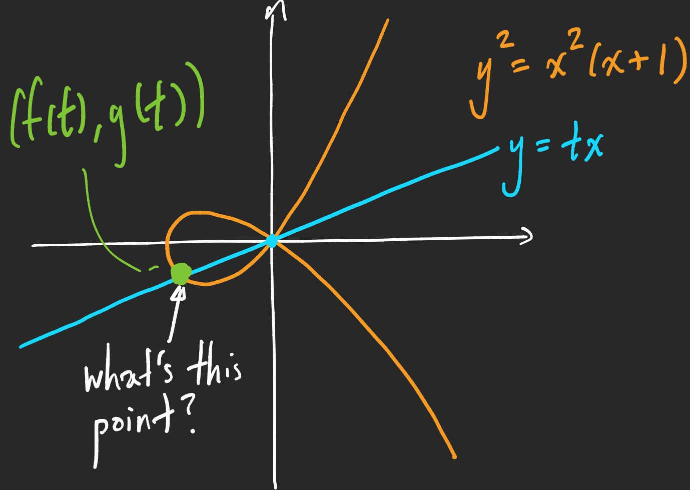


$$\begin{cases} y^2 = x^2(x+1) \\ y = tx \end{cases}$$

$$t^2x^2 = x^2(x+1)$$

$$x^2(t^2 - x - 1) = 0$$

So either $x = 0$ or $x = t^2 - 1$.

corresponds to origin

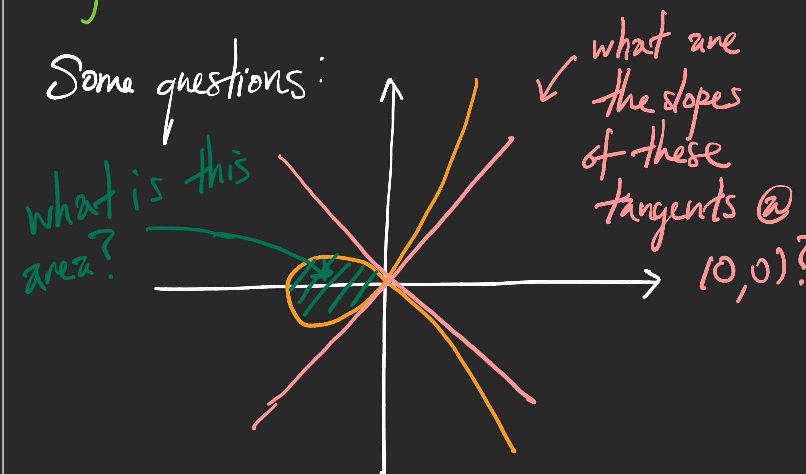
this is the one I want.

$x = t^2 - 1$ $\quad$ $y = t(t^2-1) = t^3 - t$

$f(t) = t^2 - 1$

$g(t) = t(t^2 - 1)$

Some questions:



Recall: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2 - 1}{2t}$

(provided $dx/dt \neq 0$).

What t values correspond to $(x,y) = (0,0)$?

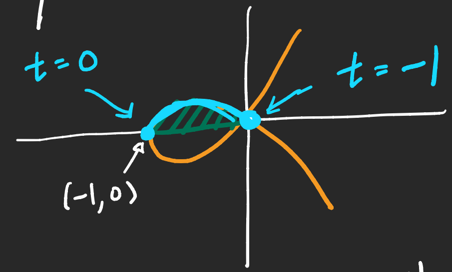
$$\begin{cases} t^2 - 1 = 0 \\ t(t^2-1) = 0 \end{cases}$$

The only solutions are $t = -1, 1$.

The slopes are: $\dfrac{2(-1)^2 - 1}{2(-1)} = -1$

and $\dfrac{2(1)^2 - 1}{2(1)} = 1$.

Area in question is 2 times



This area is $\displaystyle\int_{-1}^{0} y\, dx = \int_{0}^{-1} g(t) f'(t)\, dt$

$x = f(t)$

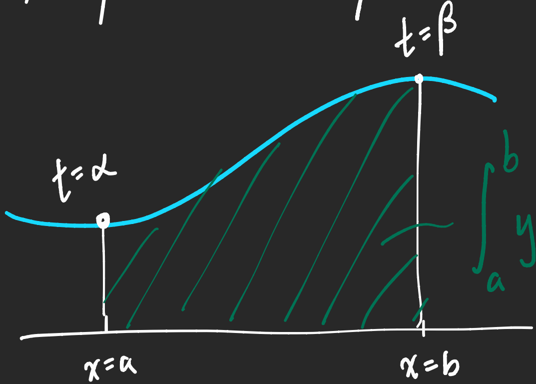
$$= \int_{0}^{-1} t(t^2-1)2t\ dt = \int_{0}^{-1} (2t^4 - 2t^2)\, dt$$

$$= \left(\frac{2}{5}t^5 - \frac{2}{3}t^3\right)\Bigg|_{t=0}^{-1} = -\frac{2}{5} + \frac{2}{3} = \frac{4}{15}$$

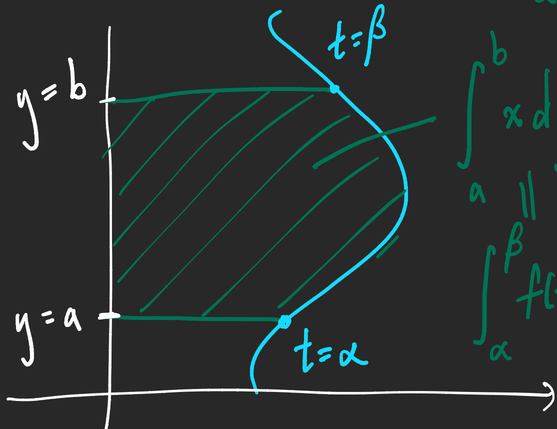
So final answer is $8/15$.

Examples of area questions

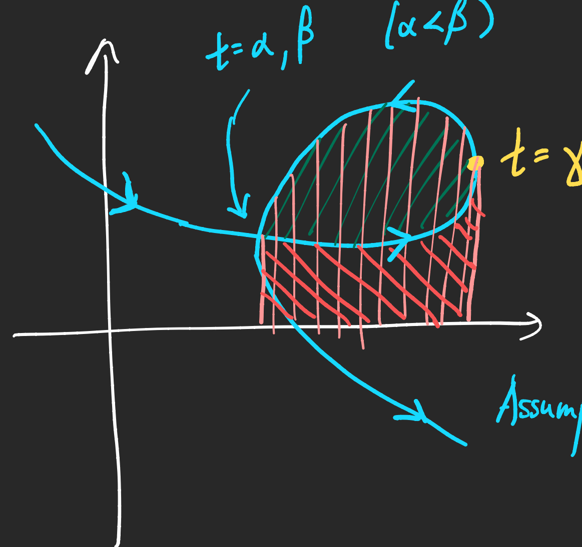
$x = f(t)$
$y = g(t)$



$$\int_a^b y\,dx = \int_\alpha^\beta g(t)f'(t)\,dt$$



$$\int_a^b x\,dy = \int_\alpha^\beta f(t)g'(t)\,dt$$



Assumption:

$$= \int_\beta^\gamma g(t)f'(t)\,dt - \int_\alpha^\gamma g(t)f'(t)\,dt$$

$$= \int_\beta^\gamma g(t)f'(t)\,dt + \int_\gamma^\alpha g(t)f'(t)\,dt$$

$$= \int_\beta^\alpha g(t)f'(t)\,dt$$

In general: the area enclosed by a parametric "loop" that starts at $t=\alpha$ and ends at $t=\beta$ is

$$= \left| \int_\alpha^\beta g(t)f'(t)\,dt \right|$$

$$= \left| \int_\beta^\alpha g(t)f'(t)\,dt \right|$$

$$= \left| \int_\alpha^\beta f(t)g'(t)\,dt \right|$$

$$= \left| \int_\beta^\alpha f(t)g'(t)\,dt \right|$$

(it turns out these are all the same. Can you show it?)